UNIT I PDE

1. From the partial Differential equations by eleminating ~~orbital~~ arbitrary constants from $z = ax + by$

Soln $z = ax + by$ — ①

Diff w.r. to 'x'

$p = a$ — ②

Diff w.r. to 'y'

$q = b$ — ③

Sub ② & ③ in equ ①

$$\boxed{z = px + qy}$$

2. From the PDE by eleminating a & b from $z = a(x+y) + b$

Soln $z = a(x+y) + b$ — ①

$ax + ay + b$,

Diff w.r. to 'x'

$p = a$ — ②

Diff w.r. to 'y'

$q = a$ — ③

Equate ② & ③ $p = q$

$p - q = 0$

3. From the PDE by eleminating a & b from $z = (x^2+a^2)(y^2+b^2)$

Soln $z = (x^2+a^2)\cdot(y^2+b^2)$ — ①

Diff w.r. to 'x'

$p = (y^2+b^2)\cdot 2x$ — ②

Diff w.r. to 'y'

$q = (x^2+a^2)\cdot 2y$ — ③

$(y^2+b^2) = p/2x$

$(x^2+a^2) = q/2y$

Sub the values in eqn ①.

$z = \frac{p}{2x} \cdot q/2y$

$\boxed{4xyz = pq}$

4. From the PDE by eleminating a & b from $z = (x+a)^2 + (y+b)^2$

Soln $z = (x+a)^2 + (y+b)^2$ —— ①

Diff. w.r. to 'x'

$p = 2(x+a) \Rightarrow x+a = p/2$ —— ②

Diff. w.r. to 'y'

$q = 2(y+b) \Rightarrow (y+b) = q/2$ —— ③

Sub the values ② & ③ in eqn ①

$z = \frac{p^2}{4} + q^2/4$

$\boxed{4z = p^2+q^2}$

5. Form the PDE by eleminating arbitary constant

$z = ax^n + by^n$

Soln $z = ax^n + by^n$ —— ①

Diff. w.r. to 'x'

$p = anx^{n-1}$

$\frac{px}{n} = ax^n$ —— ② .

Diff. w.r. to 'y'

$q = bny^{n-1}$

$$\frac{qy}{n} = by^n \quad \text{—— (3)}$$

Sub (2) & (3) in equ (1)

$$z = \frac{px}{n} + \frac{qy}{n}$$

$$\boxed{nz = px + qy}$$

6. Obtain the partial Differential equations by eleminating constands a & b from $(x-a)^2+(y-b)^2+z^2=1$

Soln $(x-a)^2+(y-b)^2+z^2=1$ —— (1)

Diff. w.r. to 'x'

$2(x-a)+2zp=0$

$\cancel{2}(x-a) = -\cancel{2}zp$

$x-a = -zp$ —— (2)

Diff. w.r. to 'y'

$2(y-b)+2zq=0$

$2(y-b) = -2zq$

$y-b = -zq$ —— (3)

Sub the (2) & (3) in equ (1)

$z^2p^2+z^2q^2+z^2=1$

$$\boxed{z^2(p^2+q^2+1)=1}$$

1. Form the PDE by eleminating the orbitrary function
$z = f(x^2+y^2) + x+y$

Soln

$z = f(x^2+y^2) + x+y$ —— ①

Diff. w.r. to 'x'

$p = f'(x^2+y^2) \cdot 2x + 1$

$p-1 = f'(x^2+y^2) \cdot 2x$ —— ②

Diff. w.r. to 'y'

$q = f'(x^2+y^2) \cdot 2y + 1$

$q-1 = f'(x^2+y^2) \cdot 2y$ —— ③

$\frac{②}{③} \Rightarrow \frac{p-1}{q-1} = \frac{x}{y}$

$(p-1)\, y = (q-1)\, x$

$py - y = qx - x$

$\boxed{py - qx = y - x}$

2. Obtain PDE by eleminating the orbiatric function
$z = f(x^2+y^2)$

Soln $z = f(x^2+y^2)$ —— ①

Diff. w.r. to 'x'

$P = f'(x^2+y^2)\,2x$ —— ②

Diff ① par w.r. to 'y'

$q = f'(x^2+y^2)\,2y$ —— ③

$\frac{②}{③} \Rightarrow \frac{p}{q} = \frac{x}{y}$

$py = qx$

$py - qx = 0$

3. Abtain PDE by eleminating the orbitatrory function

$z = f(xy)$

Soln $z = f(xy)$ —— ①

Diff w.r. to 'x'

$P = f'(xy)\,y$ —— ②

Diff w.r. to 'y'

$q = f'(xy)\,x$ —— ③

$\frac{②}{③} \Rightarrow \frac{p}{q} = \frac{y}{x}$

$\boxed{px - qy = 0}$

1 Solve $4\dfrac{\partial^2 z}{\partial x^2} - 12\dfrac{\partial^2 z}{\partial x \partial y} + 9\dfrac{\partial^2 z}{\partial y^2} = 0$

<u>Soln</u>

$$4\frac{\partial^2 z}{\partial x^2} - 12\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = 0$$

$$(4D^2 - 12DD' + 9D'^2)z = 0$$

The A.E is $4m^2 - 12m + 9 = 0$

$$4m^2 - 6m - 6m + 9 = 0$$

$$2m(2m-3) - 3(2m-3) = 0$$

$$(2m-3)(2m-3) = 0$$

$$m = 3/2,\ 3/2$$

$$C.F = \phi_1(y + 3/2\,x) + x\,\phi_2(y + 3/2\,x)$$ (Ans)

2 Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = 0$

<u>Soln</u>

$$(D^3 + D^2D' - DD'^2 - D'^3)z = 0$$

The A.E is

$$m^3 + m^2 - m - 1 = 0$$

$$m = 1$$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -1 & -1 \\ & 0 & 1 & 2 & 1 \\ \hline & 1 & 2 & 1 & \boxed{0} \end{array}$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m = -1, -1$$

$$m = -1, -1, 1$$

$$C.F = \phi_1 (y-x) + x\, \phi_2 (y-x) + \phi_3 (y+x)$$

(Ans)

3 Solve $(D^3 - 3D^2D' + 2DD'^2) z = 0$

Soln

$$(D^3 - 3D^2D' + 2DD'^2) z = 0$$

The A.E is

$$m^3 - 3m^2 + 2m = 0$$

$$m(m^2 - 3m + 2) = 0$$

$$m (m-2)(m-1) = 0$$

$$m = 0, 1, 2$$

$$C.F = \phi_1(y) + \phi_2 (y+x) + \phi_3 (y+2x)$$ (Ans)

4 $(D^4 - 2D^3D' + 2DD'^3 - D'^4)\, z = 0$

The A.E is

$$m^4 - 2m^3 + 2m - 1 = 0$$

$$\begin{array}{c|ccccc}
1 & 1 & -2 & 0 & 2 & -1 \\
 & 0 & 1 & -1 & -1 & 1 \\
\hline
 & 1 & -1 & -1 & 1 & \boxed{0}
\end{array}$$

$$m^3 - m^2 - m + 1 = 0$$

$$m = 1$$

$$\begin{array}{c|cccc}
1 & 1 & -1 & -1 & 1 \\
 & 0 & 1 & 0 & -1 \\
\hline
 & 1 & 0 & -1 & \boxed{0}
\end{array}$$

$$m^2 - 1 = 0$$

$$m^2 = 1$$

$$m = \pm 1$$

$$m = 1, 1, 1, -1$$

$$C.F = \phi_1 (y+x) + x\,\phi_2 (y+x) + x^2\,\phi_3 (y+x) + \phi_4 (y-x)$$

To find Complementary Function:

Case(i)

$$(D - mD' - \alpha)z = 0$$

$$C.F = e^{\alpha x} \phi_1 (y+mx)$$

case(ii)

$$(D - m_1 D' - \alpha_1)(D - m_2 D' - \alpha_2) z = 0$$

$$C.F = e^{\alpha_1 x} \phi_1 (y + m_1 x) + e^{\alpha_2 x} \phi_2 (y + m_2 x)$$

case(iii)

$$(D - mD' - \alpha)^r z = 0$$

$$C.F = e^{\alpha x} \phi_1 (y+mx) + x e^{\alpha x} \phi_2 (y+mx) + \dots + x^{r-1} e^{\alpha x} \phi_r (y+mx)$$

1. Solve $(D^2 - DD' + D' - 1) z = 0$

$$(D^2 - DD' + D' - 1)z = 0$$

$$D^2 - DD' + D' - 1$$

$$= \underline{D^2 - 1} - DD' + D'$$

$$(D-1)(D - D' + 1)z = 0$$

$$= (D-1)(D+1) - D'(D-1)$$

$$= (D-1)[D - D' + 1]$$

$m_1 = 0 \qquad m_2 = 1$

$\alpha_1 = 1 \qquad \alpha_2 = -1$

$$C.F = e^{x} \phi_1 (y) + e^{-x} \phi_2 (y+x)$$

NON HOMOGENEOUS PDE

To find Complementary function:

case(i)

$$(D - mD' - \alpha)z = 0$$

$$C.F = e^{\alpha x}\phi_1(y+mx)$$

case(ii)

$$(D - m_1D' - \alpha_1)(D - m_2D' - \alpha_2)z = 0$$

$$C.F = e^{\alpha_1 x}\phi_1(y+m_1x) + e^{\alpha_2 x}\phi_2(y+m_2x)$$

case(iii)

$$(D - mD' - \alpha)^r z = 0$$

$$C.F = e^{\alpha x}\phi_1(y+mx) + x e^{\alpha x}\phi_2(y+mx) + \dots + x^{r-1}e^{\alpha x}\phi_r(y+mx)$$

1. Solve $(D^2 - DD' + D' - 1)z = 0$

$$(D^2 - DD' + D' - 1)z = 0$$

$$D^2 - DD' + D' - 1$$

$$= \underline{D^2 - 1} - DD' + D'$$

$$= (D-1)(D+1) - D'(D-1)$$

$$= (D-1)[D - D' + 1]$$

$$(D-1)(D - D' + 1)z = 0$$

$m_1 = 0 \qquad m_2 = 1$

$\alpha_1 = 1 \qquad \alpha_2 = -1$

$$C.F = e^{x}\phi_1 y + e^{-x}\phi_2(y+x)$$

Solve $(D^2 - 2DD' + D'^2 - 3D + 3D' + 2)z = (e^{3x} + 2e^{-2y})^2$

Soln

$$(D^2 - 2DD' + D'^2 - 3D + 3D' + 2)z = (e^{3x} + 2e^{-2y})^2$$

Grouping method

1. Solve $(1-x)\,p + (2-y)\,q = 3-z$

$(1-x)\,p + (2-y)\,q = 3-z$

Which is Legrande's equations

$P = (1-x) \qquad Q = (2-y) \qquad R = 3-z$

The A.E is $\dfrac{dx}{P} = \dfrac{dy}{Q} = \dfrac{dz}{R}$

i.e $\dfrac{dx}{1-x} = \dfrac{dy}{2-y} = \dfrac{dz}{3-z}$ ——— ①

consider the 1^{st} & 2^{nd} ratio in ①

$$\frac{dx}{1-x} = \frac{dy}{2-y}$$

$$\frac{dx}{x-1} = \frac{dy}{y-2}$$

$$\int \frac{dx}{x-1} = \int \frac{dy}{y-2}$$

$$\log(x-1) = \log(y-2) + \log a$$

$$\log(x-1) - \log(y-2) = \log a.$$

$$\log \frac{x-1}{y-2} = \log a$$

$$\frac{x-1}{y-2} = a$$

$$u = \frac{x-1}{y-2}$$

consider the last 2 ratio's in ①

$$\frac{dy}{2-y} = \frac{dz}{3-z}$$

$$\frac{dy}{y-2} = \frac{dz}{z-3}$$

$$\int \frac{dy}{y-2} = \int \frac{dz}{z-3}$$

$$\log(y-2) = \log(z-3) + \log b$$

$$\log\left(\frac{y-2}{z-3}\right) = \log b$$

$$\frac{y-2}{z-3} = b$$

$$v = \frac{y-2}{z-3}$$

The G.S is

$$\phi\left(\frac{x-1}{y-2}, \frac{y-2}{z-3}\right) = 0 \quad \text{(Ans)}$$

Type 1:

$f(p, q) = 0$

1. Solve $p+q = pq$

$p+q = pq$ ——— ①

Let $z = ax+by+c$ ——— ② be a soln of ①

Diff p. w. r. to 'x' & 'y'

$p = a$

$q = b$

Subs these in ①

$a+b = ab$

$b - ab = -a$

$b(1-a) = -a$

$b = \frac{-a}{1-a}$

$b = a/a-1$

Subs these value in ②

$z = ax + \frac{a}{a-1} y + c$ ——— ③

which is C.I

To Find S.I (Single Integral)

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Type 2:

$z = px + qy + f(p, q)$ $\rightarrow$ Clairaut's form

1. Solve $z = px + qy + \sqrt{1+p^2+q^2}$

put $p = a$

$q = b$ in (1)

$z = ax + by + \sqrt{1+a^2+b^2}$ ——— (2)

UNIT II FOURIER SERIES

1 State dirichlet's condition's for a given function to expand in fourier series.

Soln

A function $f(x)$ is defined in $c \le x \le c+2l$. can be expanded as an infinite trignometric Series of the form

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}.$$

Provided

1) $f(x)$ is single valued and finite in $(c, c+2l)$

2) $f(x)$ is continous or piecewise continous with finite discontinous in $(c, c+2l)$

3) $f(x)$ has no or finite numbers of maxima or minima in $(c, c+2l)$

2. Does $f(x) = \tan x$ posses a fourier expansion

$\tan x$ cannot be expanded as a fourier Series since $\tan x$ not satisfied dirichlet's condition [$\tan x$ has infinite number of infinite discontinous]

3. If $f(x)$ is discontinous at $x=a$, what does its fourier series represent at that point

case(i)

$f(x)$ is discontinous at $x=a$ and $x=a$ is an endpoint $\left[\text{The fourier series of } f(x)\right]_{x=a} = \frac{f(a)+f(a+2l)}{2}$

case 2: $f(x)$ is discontinous at $x=a$ and $x=a$ is an middle point

$$\left[\text{The fourier series of } f(x)\right]_{x=a} = \frac{f(a+)+f(a-)}{2}$$

4. determine b_n in the fourier series expansion of $f(x) = \frac{1}{2}(\pi-x)$ in $0<x<2\pi$ with the Period 2π

$$f(x) = \frac{1}{2}(\pi-x), \quad 0<x<2\pi$$

$$b_n = \frac{1}{2\pi}\int_0^{2\pi}(\pi-x)\sin nx\, dx.$$

$$= \frac{1}{2\pi}\left[(\pi-x)\left(\frac{-\cos nx}{n}\right) - (-1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{n}+\frac{\pi}{n}\right]$$

$$= \frac{1}{2\pi}\times\frac{2\pi}{n}$$

$$b_n = \frac{1}{n}$$

5 If the Fourier series for the function $f(x) = \begin{cases} 0 & 0 < x < \pi \\ \sin x & \pi < x < 2\pi \end{cases}$

$$f(x) = -1/\pi + 2/\pi \left[\frac{\cos 2x}{1\cdot 3} + \frac{\cos 4x}{3\cdot 5} + \frac{\cos 6x}{5\cdot 7} + \cdots \right]$$ $\frac{1}{2}\sin nx$

find the value of $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \cdots$

$$f(x) = -1/\pi + 2/\pi \left[\frac{\cos 2x}{1\cdot 3} + \frac{\cos 4x}{3\cdot 5} + \frac{\cos 6x}{5\cdot 7} + \cdots \right]$$ $\frac{1}{2}\sin nx$

put $x = \pi/2$ in ①

which is continous point

$$-1/\pi + 2/\pi \left[-1/1\cdot 3 + 1/3\cdot 5 - 1/5\cdot 7 + \cdots \right] + 1/2 = f(\pi/2) = 0$$

$$-2/\pi \left[1/1\cdot 3 - 1/3\cdot 5 + 1/5\cdot 7 - \cdots \right] = -1/2 + 1/\pi$$

$$-\frac{2}{\pi} \left[\frac{1}{1\cdot 3} - 1/3\cdot 5 + 1/5\cdot 7 - \cdots \right] = \frac{2-\pi}{2\pi}$$

$$\frac{1}{1\cdot 3} - 1/3\cdot 5 + 1/5\cdot 7 - \cdots \Big] = \frac{2-\pi}{2\pi} \left(-\pi/2\right)$$

$$1/1\cdot 3 - 1/3\cdot 5 + 1/5\cdot 7 - \cdots = \frac{\pi - 2}{4}$$

Find a_n in expanding e^{-x} as Fourier series in $(-\pi, \pi)$

$f(x) = e^{-x}$ in $(-\pi, \pi)$

$f(x)$ is neither even nor odd.

$$a_n = 1/\pi \int_{-\pi}^{\pi} e^{-x} \cos nx \, dx.$$

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$$= \frac{1}{\pi}\left[\frac{e^{-\pi}}{1+n^2}\left[e^{-\pi}+e^{\pi}\right]\right]$$

$$a_n = \frac{1}{\pi}\frac{(-1)^n}{1+n^2}\,2.\sinh\pi$$

7. Find the constant term in the Fourier series corresponding to $f(x) = \cos^2 x$ expanded in interval $(-\pi, \pi)$

$$f(x) = \cos^2 x \quad (-\pi, \pi)$$

$f(x)$ is an even function

$$a_0 = \frac{2}{\pi}\int_0^{\pi} \cos^2 x\, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{1+\cos 2x}{2}\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} 1+\cos 2x\, dx$$

$$= \frac{1}{\pi}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$= \frac{1}{\pi} \times \pi$$

$$a_0 = 1$$

8. If the Fourier series of the function $f(x) = x^2 + x$ in the interval $-\pi < x < \pi$ is $\frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left[\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right]$ then find the values of infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \cdots$

$f(x) = x + x^2$ in $-\pi < x < \pi$

The Fourier series of $f(x)$ is given by

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2} \cos nx - 2/n \sin nx \right] \quad -(1)$$

Put $x = \pi$ in (1)

which is discontinous endpoint

$$\frac{f(\pi) + f(-\pi)}{2} = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^4 \left[\frac{4}{n^2} (-1)^n \right]$$

$$\frac{\pi + \pi^2 - \pi + \pi^2}{2} = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^{2n} 4/n^2$$

$$\pi^2 - \pi^2/3 = 4 \sum_{n=1}^{\infty} 1/n^2 (-1)^{2n}$$

$$\frac{2\pi^2}{3} = 4 \sum_{n=1}^{\infty} 1/n^2 (-1)^{2n}$$

$$\left[1/1^2 + 1/2^2 + 1/3^2 + \dots \right] = \pi^2/6.$$

what is the constant term a_0 and co-efficient of $\cos nx$, a_n in the fourier series expansion of $f(x) = x - x^3$ in $(-\pi, \pi)$

$$f(x) = x - x^3 \quad (-\pi, \pi)$$

$$f(-x) = -x - (-x)^3 = -(x - x^3)$$

$$f(-x) = -f(x)$$

Hence $f(x)$ is an odd function $a_0 = 0$ & $a_n = 0$

10. In the fourier series expansion of $f(x) = \begin{cases} 1+\frac{2x}{\pi} & -\pi < x < 0 \\ 1-\frac{2x}{\pi} & 0 < x < \pi \end{cases}$ find the value of b_n and co-efficient of $\sin nx$

Soln

$$f(x) = \begin{cases} 1+\frac{2x}{\pi}, & -\pi < x < 0 \\ 1-\frac{2x}{\pi}, & 0 < x < \pi \end{cases}$$

$$f(-x) = \begin{cases} 1-\frac{2x}{\pi}, & -\pi < -x < 0 \\ 1+\frac{2x}{\pi}, & 0 < -x < \pi \end{cases}$$

$$= \begin{cases} 1-2x/\pi, & 0 < x < \pi \\ 1+2x/\pi, & -\pi < x < 0 \end{cases}$$

$$f(-x) = f(x)$$

$f(x)$ is even $b_n = 0$

11 Find a_0 is expanding e^{-x} as fourier series in $(-\pi,\pi)$

$f(x) = e^{-x}$ in $(-\pi,\pi)$

$f(-x) = e^{x}$

$f(-x) \neq -f(x)$

$f(-x) \neq +f(x)$

$f(x)$ is neither even nor odd.

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} e^{-x}\,dx$$

$$= \frac{1}{\pi} \left[\frac{e^{-x}}{-1} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left[e^{-\pi} + e^{+\pi} \right] = \frac{1}{\pi} 2 \sinh \pi$$

. Find fourier constant b_n for $x \sin x$ in $(-\pi,\pi)$

$f(x) = x \sin x$ in $(-\pi,\pi)$

$f(-x) = (-x) \sin -x$

$= x \sin x$

$f(-x) = f(x)$

$f(x)$ is even

$b_n = 0$

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13 Find the value of a_n in the cosine series expansion if $f(x) = k$ in the interval $(0,10)$

$f(x)$ k in $(0,10)$

$l = 10$

$$a_n = \frac{2}{l}\int_0^{10} f(x) \cos\frac{n\pi x}{l}\, dx.$$

$$= \frac{2}{10}\int_0^{10} k \cos\frac{n\pi x}{l}\, dx.$$

$$= \frac{k}{5}\left[\frac{\sin n\pi x/l}{n\pi x/l}\right]_0^{10}$$

$$= \frac{k}{5}[0-0]$$

$$a_0 = 0$$

14 Define root means square value of a function $f(x)$ in $a < x < b$

The Rms value of a function

$y = f(x)$ in $a < x < b$ defined as

$$\bar{y} = \sqrt{\frac{\int_a^b (f(x))^2\, dx}{b-a}}.$$

15 Find the Rms value of the function $f(x) = x$ in interval $(0, l)$

$$\bar{y} = \sqrt{\frac{\int_a^b (f(x))^2\, dx}{b-a}}.$$

here $f(x) = x$ in $(0, l)$

$a = 0$

$b = l$

The rms value is $= \sqrt{\frac{\int_0^l x^2 dx}{l}}$

$= \sqrt{\frac{1}{l}\left(\frac{x^3}{3}\right)_0^l} \Rightarrow \sqrt{\frac{1}{l} \times \frac{l^3}{3}}$

$= \sqrt{\frac{l^2}{3}} = \frac{l}{\sqrt{3}}$

$\bar{y} = \frac{l}{\sqrt{3}}$

16 Find the Rms value of $f(x) = x^2$ in the interval.

$(0, \pi)$

Rms value $\bar{y} = \sqrt{\frac{\int_a^b (f(x))^2 dx}{b-a}}$

$a = 0 \qquad b = \pi$

$\bar{y} = \sqrt{\int_0^\pi \frac{x^4}{\pi}} = \sqrt{\frac{1}{\pi}\left(\frac{x^5}{5}\right)_0^\pi}$

$= \sqrt{\frac{1}{\pi} \times \frac{\pi^5}{5}}$

$\bar{y} = \frac{\pi^2}{\sqrt{5}}$

17 State parseval's identity for full range expansion of $f(x)$ as fourier series in $(0, 2l)$

If $f(x)$ is a periodic function with periodic $2l$ defined in $(0, 2l)$ then.

$$\frac{1}{l}\int_0^{2l}[f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

18 State parseval's theorem on fourier series state Parseval's identity as fourier series.

If $y = f(x)$ can be expanded as a fourier series of the form

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

in $(c, c+2l)$ then the root mean square value $\bar{y}$ of $y = f(x)$ in $(c, c+2l)$ is

$$\bar{y}^2 = \frac{1}{4}a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

UNIT III

APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

ONE DIMENTIONAL WAVE EQUATION

Write the PDE Governing one dimentional wave equation?

The one Dimensional wave equation is

$$y_{tt} = a^2 y_{xx}.$$

1. What is the constant a^2 in the wave equation $u_{tt} = a^2 u_{xx}$ or in the wave equation $\frac{\partial^2 y}{\partial t^2} = \frac{c^2 \partial^2 y}{dx^2}$ what does c^2 stands for.

$$a^2 = T/m = \frac{\text{Tension}}{\text{mass per unit length of the string}}$$

2. What are the possible solution of 1 D wave equation

(i) $y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat)$

(ii) $y(x,t) = (A_1 x + A_2)(A_3 t + A_4)$

(iii) $y(x,t) = (A_5 e^{px} + A_6 e^{-px})(A_7 e^{apt} + A_8 e^{-apt})$

3. Write the initial condition of the wave equation if the string has an initial displacement or write the initial condition of the wave equation if the string

has an initial displacement but non initial velocity.

$\frac{\partial u}{\partial t}(x,0) = 0$ for $0 < x < l$

$y(x,0) = f(x)$, $0 < x < l$

4. Write the boundary and initial condition for solvings, the vibration of string equation if the string is subjected to initial displacement $f(x)$ and initial velocity $g(x)$

Boundary conditions

$y(0,t) = 0 \quad \forall\ t \geq 0$

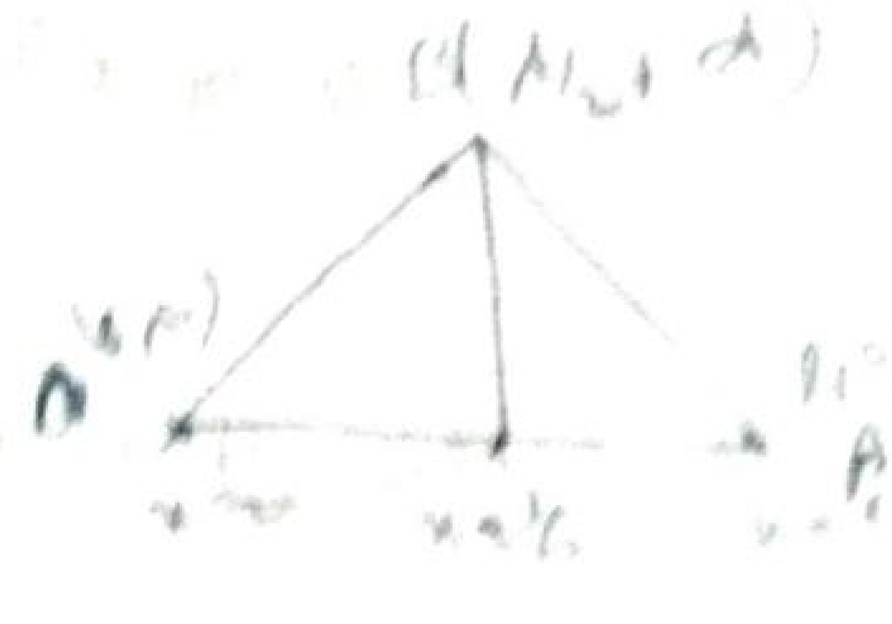
$y(l,t) = 0 \quad \forall\ t \geq 0$

Initial conditions

$\frac{\partial y}{\partial t}(x,0) = g(x)$, $0 \leq x \leq l$

$y(x,0) = f(x)$, $0 \leq x \leq l$.

5. The ends of a string of length l fixed at both ends. The midpoint of the string is taken to a height d and then released from rest. Write the initial conditions of the string.



$\frac{\partial y}{\partial t}(x,0) = 0$ $\quad 0 \leq x \leq l$

$$y(x,0) = \begin{cases} \frac{2d}{l} \cdot x & \text{in } 0 < x < l/2 \\ \frac{2d}{l}(l-x) & \text{in } l/2 < x < l \end{cases}$$

has an initial displacement but non initial velocity.

$$\frac{\partial u}{\partial t}(x,0) = 0 \quad \text{for} \quad 0 < x < l$$

$$y(x,0) = f(x) \quad , \quad 0 < x < l$$

4. Write the boundary and initial condition for solving, the vibration of string equation if the string is subjected to initial displacement $f(x)$ and initial velocity $g(x)$

Boundary conditions

$$y(0,t) = 0 \quad \forall\ t \geq 0$$

$$y(l,t) = 0 \quad \forall\ t \geq 0$$

Initial conditions

$$\frac{\partial y}{\partial t}(x,0) = g(x) \quad , \quad 0 \leq x \leq l$$

$$y(x,0) = f(x) \quad , \quad 0 \leq x \leq l$$

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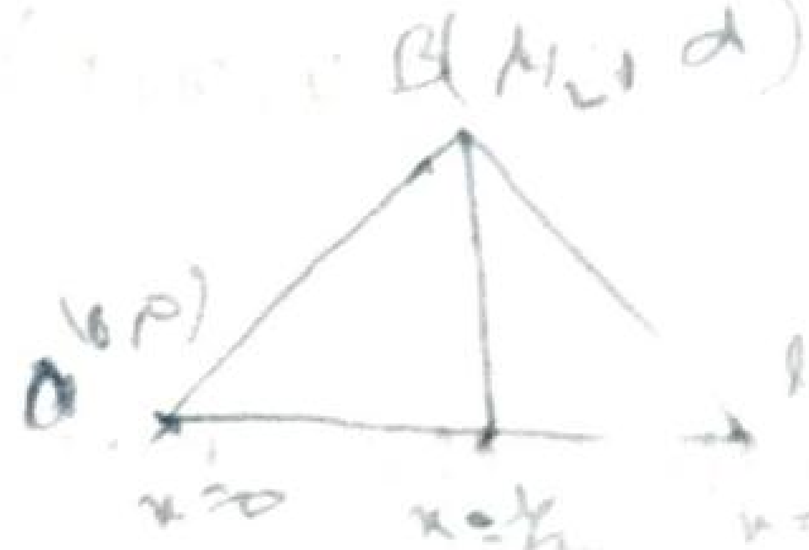

$$\frac{\partial y}{\partial t}(x,0) = 0 \quad 0 \leq x \leq l$$

$$y(x,0) = \begin{cases} \frac{2d}{l}\,x & \text{in } 0 < x < l/2 \\ \frac{2d}{l}(l-x) & \text{in } l/2 < x < l \end{cases}$$

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ONE DIMENTIONAL HEAT EQUATION

1) In steady state condition, derive the solution of one dimensionsional heat flow equation.

$$u_t = \alpha^2 u_{xx} \quad \text{---} \; (*)$$

In steady state, $u_t = 0 \Rightarrow u_{xx} = 0$

Integrate w. r. to 'x'

$$u_x = a$$

Integrate w. r. to 'x'

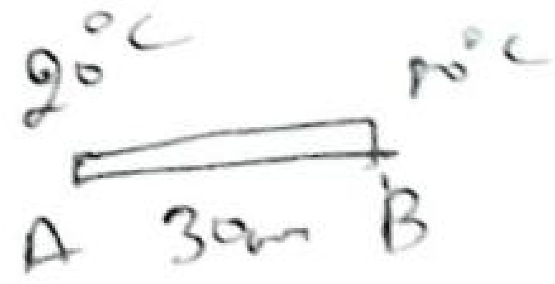
$$u = ax + b.$$

2) How many conditions are required to solve.

$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$. Three.

3) A rod 30 cm long has the ends A and B kept at $20^\circ C$ & $80^\circ C$ respectively until steady state condition prevail. Find the steady state solution of the rod.

steady state solution $u(x) = \left(\frac{b-a}{l}\right)x + a$



$b = 80^\circ c$, $a = 20^\circ c$, $l = 30cm$

$u(x) = 2x + 20$.

4) A rod 20cm long with insulated sides has its ends A and B kept at $30^\circ c$ and $90^\circ c$ respectively find the steady state temperature distribution of the rod.

steady state solution $u(x) = \left(\frac{b-a}{l}\right)x + a$

$b = 90^\circ c$, $a = 30^\circ c$, $l = 20cm$

$u(x) = 3x + 30$

5) An insulated rod of length 60cm has its ends at A and B maintained at $20^\circ c$ and $80^\circ c$ respectively. Find the steady state solution of the rod.

steady state solution $u(x) = \left(\frac{b-a}{l}\right)x + a$

$b = 80^\circ c$, $a = 20^\circ c$, $l = 60cm$

$u(x) = x + 20$.

6) Write the PDE governing 1D heat flow equation.

The 1D heat flow equation is

$u_t = \alpha^2 u_{xx}$.

7) In a diffusion equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ what does α^2 stand for.

$\alpha^2 = \frac{k}{s\rho}$ called diffusivity of the material of the bar.

$k \rightarrow$ Thermal conductivity of the material

$s \rightarrow$ specific heat capacity of the material.

$\rho \rightarrow$ density of the material.

8) Write all variable seperable soln of heat equation $u_t = \alpha^2 u_{xx}$.

$y(x,t) = (A\cos px + B\sin px)\, e^{-\alpha^2 p^2 t}$

$y(x,t) = (A_1 x + A_2)\ A_3$

$y(x,t) = (A_4\, e^{px} + A_5 e^{-px})\ (A_6 e^{-\alpha^2 p^2 t})$

9) State any two laws which are assumed to deried one D ~~each~~ heat equation.

(i) Heat flows from higher to lower temperature

(ii) The rate at which heat flows across any area is propotional to the area and to the temperature gradient normal to the curve This constant of propotionally is known as the thermal conductivity (k) of the material. ~~It~~ is known as fourier laws.

of heat equation.

v) What is the basic differenc between the solution of 1D wave equation and 1D heat equation.

The Suitable Solution of 1D wave equation is peridic in nature. But the suitable solution of 1D heat equation is not peridic in nature

The suitable solution of 1D heat equation contains exponential term. But the suitable solution of 1 Dimentional wave equation does not contain exponential term.

vi) State 1D heat equation with initial and boundary condition.

$u(0,t) = 0$, $\cancel{0 < x < l}$ $t \geq 0$

$u(l,t) = 0$ $\cancel{0 \leq x \leq l}$ $t \geq 0$

$0 < x \leq l$.

$u(x,0) = 0$

TWO DIMENTIONAL HEAT FLOW EQUATION

1. Write the PDE governing 2D heat flow equation:

$$\frac{\partial u}{\partial t} = \alpha^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

2. What is the 2D heat flow equation in steady state.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

3. Write dow all posibile soln of two dimentional heatflow equation in steady state (laplace equatio

(i) $u(x,y) = (A\cos px + B\sin px)\,(C\,e^{py} + De^{-py})$

(ii) $u(x,y) = (Ae^{px} + Be^{-px})\,(C\cos py + D\sin py)$

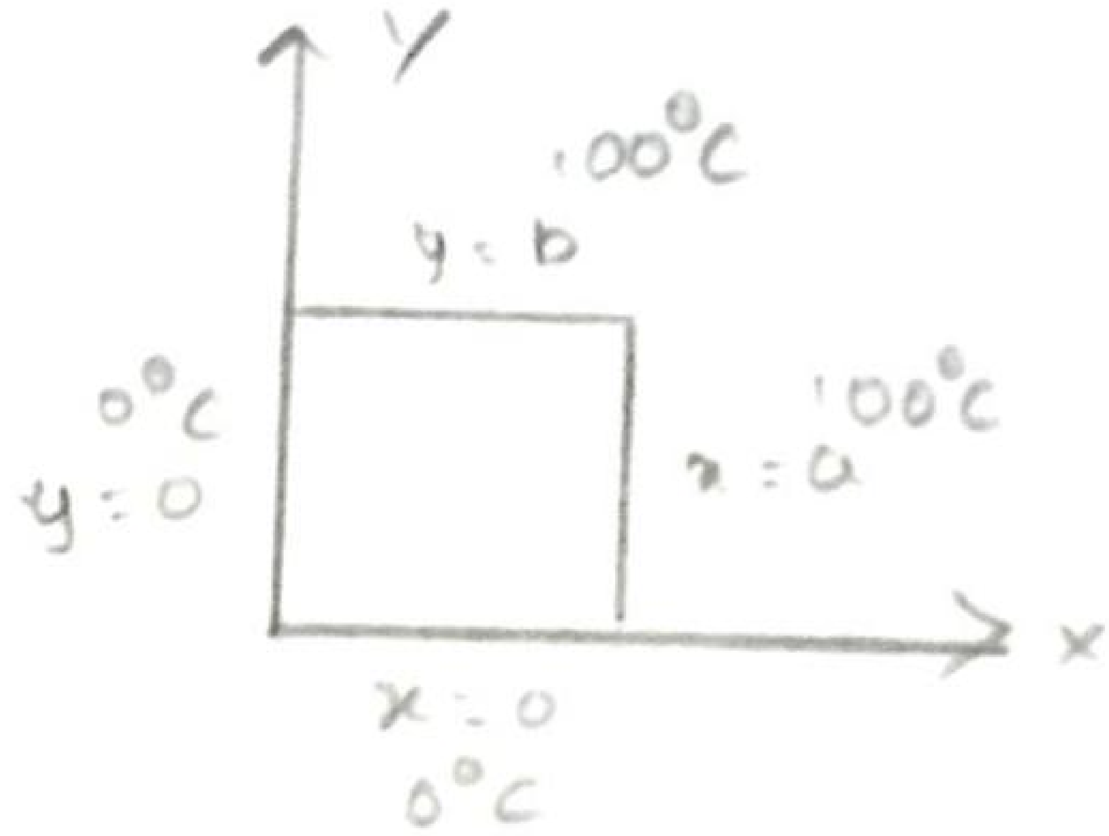
(iii) $u(x,y) = (Ax + B)\,(Cy + D)$

4. Write the 2D heatflow equation in steady state for polar quardinates

$$r^2 \frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$$

5. A rectangular plate is bounded by $x=0$ $y=0$, $x=a$, $y=b$ is surface is insulated by temperature along two edges $x=a$ and $y=b$ of $100°C$ while

the temperature along the other edges are zero °c write down the boundary condition in Mathematical form.



(i) $y(x,0) = 0°c \quad 0 \le x \le a.$

(ii) $y(x,b) = 100°c, \quad 0 \le x \le a$

(iii) $y(0,y) = 0°c, \quad 0 \le y \le b$

(iv) $y(a,y) = 100°c, \quad 0 \le y \le b.$

Write down the various possible solution of 2D heat flow equation in steady state for polar Quardinates.

$$u(r,\theta) = (A_1 r^{\lambda} + B_1 r^{-\lambda})(C_1 \cos\lambda\theta + D_1 \sin\lambda\theta)$$

$$u(r,\theta) = (A_2 e^{\lambda\theta} + B_2 e^{-\lambda\theta})[C_2 \cos(\lambda \log r) + D_2 \sin(\lambda \log r)]$$

$$u(r,\theta) = (A_3 \log r + B_3)(C_3\theta + D_3)$$

classify the one Dimentional heat equation

$u_t = \alpha^2 u_{xx}$

$\alpha^2 u_{xx} - u_t = 0$

Here $A = \alpha^2$ $B = 0$ $C = 0$

$B^2 - 4AC = 0$

$\therefore$ 1 D heat equation (or) heat conduction is parabolic.

classify one Dimentional wave equation

$u_{tt} = \alpha^2 u_{xx}$

$\alpha^2 u_{xx} - u_{tt} = 0$

Here $A = \alpha^2$ $B = 0$ $C = -1$

$B^2 - 4AC = 0$

$4\alpha^2 > 0$

$\therefore$ 1 D wave equation is hyperbolic.

classify the partial Differential equation

$3u_{xx} + 4u_{yy} + 3u_y - 2u_x = 0$

$A = 3$ $B = 0$ $C = 4$

$B^2 - 4AC = -48 < 0$

$\therefore$ paritial Differential Equation is elliptic.

classify the following Partial Differential equation

a) $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$

$A = 1 \quad B = 0 \quad C = -1$

$B^2 - 4AC = 4 > 0$

$\therefore$ The PDE is hyperbolic.

b) $$\frac{\partial^2 u}{\partial x \partial y} = \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) + xy$$

$A = 0 \quad B = 1 \quad C = 0$

$B^2 - 4AC = 1 > 0$

The given PDE is hyperbolic.

Classify the following PDE is

(i) $$4\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} - 6\frac{\partial u}{\partial x} - 8\frac{\partial u}{\partial y} - 16u = 0$$

$A = 4 \quad B = 4 \quad C = 1$

$B^2 - 4AC = 16 - 16 = 0$

$\therefore$ The given equation is parabolic.

(ii) $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$$

$A = 1 \; ; B = 0 \; ; C = 1$

$B^2 - 4AC = -4 < 0$

$\therefore$ The given equation is elliptic

classify the PDE

a) $y^2 u_{xx} - 2xy\, u_{xy} + x^2 u_{yy} + 2u_x - 3u = 0$

$A = y^2 \quad B = -2xy \quad C = x^2$

$B^2 - 4AC = 4x^2y^2 - 4x^2y^2 = 0$

$\therefore$ The given equation is Parabolic.

b) $y^2 u_{xx} + u_{yy} + u_x^2 + u_y^2 + 7 = 0$

$A = y^2 \quad B = 0 \quad C = 1$

$B^2 - 4AC = -4y^2$

case (i) $y = 0$

$B^2 - 4AC = 0$

The given equation is parabolic.

case (ii) $y \neq 0$

$B^2 - 4AC < 0$

The given equation is elliptic.

$x^2 z_{xx} + (1-y^2) z_{yy}$

$B^2 - 4AC = 0$

$A = x^2$

$B = 0$

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$C = 1 - y^2$

$B^2 - 4AC = -4x^2(1-y^2)$

$= 4x^2(y^2-1)$

If $x = 0$ $B^2 - 4AC = 0$

The given equation is parabolic

If $x \neq 0$ x^2 is +ve

case (i)

$y = +1$ (or) -1

$B^2 - 4AC = 0$

$\therefore$ The given equation is parabolic

case (ii)

$-1 < y < 1$

$B^2 - 4AC < 0$

Here the equation is ellipitic

case (iii) $y > 1$ & $y < -1$

$B^2 - 4AC > 0$

Here given equation is hyperbolic.

$C = 1 - y^2$

$B^2 - 4AC = -4x^2(1-y^2)$

$= 4x^2(y^2-1)$

If $x = 0$ $B^2 - 4AC = 0$

The given equation is parabolic

If $x \neq 0$ x^2 is +ve

case (i)

$y = +1$ (or) -1

$B^2 - 4AC = 0$

$\therefore$ The given equation is parabolic

case (ii)

$-1 < y < 1$

$B^2 - 4AC < 0$

Here the equation is elliptic

case (iii) $y > 1$ & $y < -1$

$B^2 - 4AC > 0$

Here given equation is hyperbolic.

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UNIT IV FOURIER TRANSFORM

1.Find the fourier sine transform of $1/x$

$$F_s[f(x)] = \sqrt{2/\pi} \int_0^\infty f(x) \sin sx \, dx$$

$$= \sqrt{2/\pi} \int_0^\infty 1/x \sin sx \, dx$$

Put $sx = \theta$

$x = \theta/s$

$dx = d\theta/s$

$$= \sqrt{2/\pi} \int_0^\infty \cancel{s}/\theta \sin\theta \, d\theta/\cancel{s}$$

$$= \sqrt{2/\pi} \int_0^\infty \frac{\sin\theta}{\theta} d\theta$$

$$= \sqrt{2/\pi} \; \pi/2$$

$$F_s[f(x)] = \sqrt{\pi/2}$$

2.Find the fourier cosine transform of $f(x) = \begin{cases} \cos x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

$$F_c[f(x)] = \sqrt{2/\pi} \int_0^\infty f(x) \cos sx \, dx$$

$$= \sqrt{2/\pi} \int_0^1 \cos x \cos sx \, dx$$

$$= \sqrt{2/\pi} \int_0^1 \cos sx \cos x \, dx.$$

$$= \sqrt{2/\pi}\ \frac{1}{2} \int_0^1 \cos(s+1)x + \cos(s-1)x\, dx$$

$$= \frac{1}{2}\sqrt{2/\pi} \left[\frac{\sin(s+1)x}{s+1} + \frac{\sin(s-1)x}{s-1} \right]_0^1$$

$$= \frac{1}{2}\sqrt{2/\pi} \left[\frac{\sin(s+1)}{s+1} + \frac{\sin(s-1)}{s-1} \right]$$

$$F_c[f(x)] = \frac{1}{2}\sqrt{2/\pi} \left[\frac{\sin(s+1)}{s+1} + \frac{\sin(s-1)}{s-1} \right]$$

3. Find the fourier sine transform of

$$f(x) = \begin{cases} \sin x & ; \quad 0 < x < a \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$F_s[f(x)] = \sqrt{2/\pi} \int_0^\infty f(x) \sin sx\, dx$$

$$= \sqrt{2/\pi} \int_0^a \sin x \sin sx\, dx$$

$$= \sqrt{2/\pi}\ \frac{1}{2} \int_0^a \cos(s-1)x - \cos(s+1)x$$

$$F_s[f(x)] = \frac{1}{2}\sqrt{2/\pi} \left[\frac{\sin(s-1)x}{s-1} - \frac{\sin(s+1)x}{s+1} \right]_0^a$$

$$F_s[f(x)] = \frac{1}{2}\sqrt{2/\pi} \left[\frac{\sin(s-1)a}{s-1} - \frac{\sin(s+1)a}{s+1} \right]$$

4. Find the fourier sine transform

$$f(x) = \begin{cases} \sin x & , \quad 0 < x < \pi \\ 0 & , \quad \pi < x < \infty \end{cases}$$

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$$F_s[f(x)] = \sqrt{2/\pi} \int_0^\infty f(x) \sin sx \, dx$$

$$= \sqrt{2/\pi} \int_0^\pi \sin sx \sin x \, dx$$

$$= \frac{1}{2}\sqrt{2/\pi} \int_0^\pi \cos(s-1)x - \cos(s+1)x \, dx$$

$$= \frac{1}{2}\sqrt{2/\pi} \left[\frac{+\sin(s-1)x}{s-1} - \frac{\sin(s+1)x}{s+1} \right]_0^\pi$$

$$= \frac{1}{2}\sqrt{2/\pi} \left[\frac{+\sin(s-1)\pi}{s-1} - \frac{\sin(s+1)\pi}{s+1} \right]$$

$$= \frac{1}{2}\sqrt{2/\pi} \left[-\frac{\sin(s+1)\pi}{s+1} + \frac{\sin(s-1)\pi}{s-1} \right]$$

$$= \frac{1}{2}\sqrt{2/\pi} \sin s\pi \left[-\frac{1}{s+1} - \frac{1}{s-1} \right]$$

$$= \frac{1}{\cancel{2}}\sqrt{2/\pi} \sin s\pi \left[\frac{-\cancel{2}}{s^2-1} \right]$$

$$F_s[f(x)] = \sqrt{2/\pi} \left[\frac{\sin s\pi}{1-s^2} \right]$$

Find the fourier sine transform of $f(x) = 1$ $(0, l)$

$$F_s[f(x)] = \sqrt{2/\pi} \int_0^\infty f(x) \sin sx \, dx$$

$$= \sqrt{2/\pi} \int_0^l \sin sx \, dx.$$

$$= \sqrt{2/\pi} \left[\frac{-\cos sx}{s} \right]_0^l$$

$$F_s [f(x)] = \sqrt{2/\pi} \left[\frac{-\cos ls}{s} + 1/s \right]$$

$$F_s [f(x)] = \sqrt{2/\pi} \left[\frac{1 - \cos sl}{s} \right]$$

6. Find the function $f(x)$ to sin transform is e^{-as}

$$F_s (s) = e^{-as}$$

$$f(x) = \sqrt{2/\pi} \int_0^\infty F_s (s) \sin sx \, ds$$

$$= \sqrt{2/\pi} \int_0^\infty e^{-as} \sin sx \, dx$$

~~diff w. x to s~~

$$= \sqrt{2/\pi} \left[\frac{e^{-as}}{a^2+x^2} (-a \sin sx - x \cos sx) \right]_0^\infty$$

$$= \sqrt{2/\pi} \left[\frac{x}{a^2+x^2} \right]$$

$$f(x) = \sqrt{2/\pi} \left[\frac{x}{a^2+x^2} \right]$$

7. Find Fourier sine transform of $f(ax)$

$$F_s[f(ax)] = \sqrt{2/\pi} \int_0^\infty f(ax) \sin sx \, dx$$

put $ax = t$

$x = t/a$

$dx = dt/a$

$$= \sqrt{2/\pi} \int_0^\infty f(t) \sin s\, t/a \; dt/a$$

$$= 1/a \sqrt{2/\pi} \int_0^\infty f(t) \sin s/a \, t \; dt$$

$$F_s[f(ax)] = 1/a \; F_s[s/a]$$

8. Find fourier cosine transform of $f(ax)$

$$F_c[f(ax)] = \sqrt{2/\pi} \int_0^\infty f(ax) \cos sx \, dx$$

put $ax = t$

$x = t/a$

$dx = dt/a$

$$= \sqrt{2/\pi} \int_0^\infty f(t) \cos s\, t/a \; dt/a$$

$$= 1/a \sqrt{2/\pi} \int_0^\infty f(t) \cos s/a \; dt$$

$$F_c[f(ax)] = 1/a \; F_c[s/a]$$

9. Find Fourier cosine transform of [0, a/2 ...

$$F_c[f(x)] = \sqrt{2/\pi} \int_0^\infty f(x) \cos sx \, dx$$

$$= \sqrt{2/\pi} \int_0^a \cos x \cos sx \, dx$$

$$= \sqrt{2/\pi} \int_0^a \cos sx \cos x \, dx$$

$$= \sqrt{2/\pi} \, \frac{1}{2} \int_0^a \cos(s+1)x + \cos(s-1)x \, dx$$

$$= \frac{1}{2} \sqrt{2/\pi} \left[\frac{\sin(s+1)x}{s+1} + \frac{\cos(s-1)x}{s-1} \right]_0^a$$

$$F_c[f(x)] = \frac{1}{2} \sqrt{2/\pi} \left[\frac{\sin(s+1)a}{s+1} + \frac{\sin(s-1)a}{s-1} \right]$$

10. Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0<x<1 \\ 1, & 1<x<2 \\ 0, & x>2 \end{cases}$

$$F_s[f(x)] = \sqrt{2/\pi} \int_0^\infty f(x) \sin sx \, dx$$

$$= \sqrt{2/\pi} \left[\int_0^1 x \sin sx \, dx + \int_1^2 \sin sx \, dx \right]$$

$$= \sqrt{2/\pi} \left[x\left(\frac{-\cos sx}{s}\right) - \left(\frac{-\sin sx}{s^2}\right) \right]_0^1 + \left[-\frac{\cos sx}{s} \right]$$

$$= \sqrt{2/\pi} \left[\cancel{\frac{-\cos s}{s}} + \frac{\sin s}{s^2} - \frac{\cos 2s}{s} + \cancel{\frac{\cos s}{s}} \right]$$

$$= \sqrt{2/\pi} \left[\frac{\sin s}{s^2} - \frac{\cos 2s}{s} \right]$$

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11. Find the fourier sine transform of $F_s[f(x)\cos ax]$

Proof

$$F_s[f(x)\cos ax] = \sqrt{2/\pi}\int_0^\infty f(x)\cos ax \sin sx \, dx$$

$$= \sqrt{2/\pi}\int_0^\infty f(x)\sin sx \cos ax \, dx$$

$$= \sqrt{2/\pi}\int_0^\infty f(x)\left[\frac{\sin(s+a)x + \sin(s-a)x}{2}\right]dx$$

$$= \frac{1}{2}\left[\sqrt{2/\pi}\int_0^\infty f(x)\sin(s+a)x \, dx + \sqrt{2/\pi}\int_0^\infty f(x)\sin(s-a)x \, dx\right]$$

$$F_s[f(x)\cos ax] = \frac{1}{2}\left[F_s(s-a) + F_s(s+a)\right]$$

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12. Find the sine transform of $f(x)\sin ax$

$$F_s[f(x)\sin ax] = \frac{1}{2}\left[F_c(s-a) - F_c(s+a)\right]$$

$$F_s[f(x)\sin ax] = \sqrt{2/\pi}\int_0^\infty f(x)\sin ax \sin sx \, dx$$

$$= \sqrt{2/\pi}\int_0^\infty f(x)\sin sx \sin ax \, dx$$

$$= \sqrt{2/\pi}\int_0^\infty f(x)\left[\frac{\cos(s-a)x - \cos(s+a)x}{2}\right] dx$$

$$= \frac{1}{2}\left[\sqrt{2/\pi}\int_0^\infty f(x)\cos(s+a)x - \sqrt{2/\pi}\int_0^\infty f(x)\cos(s- \right.$$
$x\,d$

$$F_s[f(x)\sin ax] = \frac{1}{2}\left[F_c(s-a) - F_c(s+a)\right]$$

13. Find the fourier cosine transform of $f(x)\sin ax$

$\cancel{\frac{1}{2}\left[F_s(s-a) - F_s(s+a)\right]}$

$$F_c[f(x)\sin ax] = \sqrt{2/\pi}\int_0^\infty f(x)\sin ax \cos sx \, dx$$

$$= \sqrt{2/\pi}\int_0^\infty f(x)\left[\frac{\sin(s+a)x + \sin(s+a)x}{2}\right] dx$$

$$= \frac{1}{2}\left[\sqrt{2/\pi}\int_0^\infty f(x)\sin(s+a)x + \sqrt{2/\pi}\int_0^\infty f(x)\sin(\right.$$

$$F_c[f(x)\sin ax] = \frac{1}{2}\left[F_s(s+a) - F_s(s+a)\right]$$

1. Find the fourier cosine transform of

$F_c[f(x)\cos ax] = 1/2[F_c \sin(s-a) + F_c \sin(s+a)]$

$F_c[f(x)\cos ax] = \sqrt{2/\pi}\int_0^\infty f(x)\cos ax \cos sx\, dx$

$= \sqrt{2/\pi}\int_0^\infty f(x)\cos sx \cos ax\, dx.$

$= \sqrt{2/\pi}\int_0^\infty f(x)\left[\frac{\cos(s-a)x + \cos(s+a)x}{2}\right] dx$

$= 1/2\left[\sqrt{2/\pi}\int_0^\infty f(x)\cos(s-a)x\, dx + \sqrt{2/\pi}\int_0^\infty f(x)\cos(s+a)x\, dx\right]$

$F_c[f(x)\cos ax] = 1/2[F_c(s-a) + F_c(s+a)]$

ii. Find the fourier cosine transform of $F_c[x f(x)]$

$F_s[f(x)] = \sqrt{2/\pi}\int_0^\infty f(x)\sin sx\, dx$

Diff. w. r. to s once.

$\frac{d}{ds} F_s[f(x)] = \sqrt{2/\pi}\int_0^\infty f(x)\cos sx \; x\, dx$

$= \sqrt{2/\pi}\int_0^\infty x f(x)\cos sx\, dx$

$= F_c[x f(x)]$

$F_c[x f(x)] = d/ds\; F_s[f(x)]$

UNIT V Z-TRANSFORMS

1. Z-transform of some basic functions

(i) If the unit impulse sequence.

$$\delta(n) = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{if } n \neq 0 \end{cases}$$

$$\delta(n-1) = \begin{cases} 1 & \text{i} \\ 0 & \end{cases}$$

$$\delta(n-2) = \begin{cases} 1 & n \\ 0 & n \end{cases}$$

$$z[\delta(n)] = 1$$

$$z[\delta(n)] =$$

$$z[f(n)] = \sum_{n=0}^{\infty} f(n) z^{-n}.$$

$$z[\delta(n-1)]$$

$$z[\delta(n-2)]$$

2. Find the z-transform of unit step sequence

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

$$z[u(n)] = \sum_{n=0}^{\infty} u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} z^{-n}$$

$$= 1 + 1/z + 1/z^2 + \dots$$

$$= (1 - 1/z)^{-1}$$

$$= \left[\frac{z-1}{z}\right]^{-1} = \frac{z}{z-1}$$

3. Find the z-transform of k

$$z[k] = \sum_{n=0}^{\infty} k z^{-n}$$

$$= k \left[1 + 1/z + 1/z^2 + \dots\right]$$

$$z[k] = \frac{kz}{z-1}$$

Result:

$$z[1] = \frac{z}{z-1}$$

$$z[2] = \frac{2z}{z-1}$$

$$z[3] = \frac{3z}{z-1}$$

4. Find the z-transform of a^n.

$$z[a^n] = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (a/z)^n$$

$$= 1 + a/z + (a/z)^2 + \dots$$

$$= (1 - a/z)^{-1}$$

$$= \left[\frac{z-a}{z}\right]^{-1}$$

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$$\boxed{z[a^n] = \frac{z}{z-a}}$$

5. Find the z- transform of e^{at}

$$z[e^{at}] = z[e^{anT}]$$

$$= z[(e^{aT})^n]$$

$$= \frac{z}{z-e^{aT}} \quad \left[\because z[a^n] = \frac{z}{z-a}\right]$$

(or)

$$z[e^{at}] = \sum_{n=0}^{\infty} e^{anT} z^{-n}$$

$$= \cancel{\sum_{n=0}^{\infty} \frac{z^{-n}}{e^{-anT}}} = \sum_{n=0}^{\infty} \frac{(e^{aT})^n}{z^n}$$

$$= \sum_{n=0}^{\infty} \cancel{\left(\frac{z}{e^{aT}}\right)^{-n}} \quad \left(\frac{e^{aT}}{z}\right)^n$$

$$= 1 + \frac{e^{aT}}{z} + \left(\frac{e^{aT}}{z}\right)^2 + \ldots\ldots$$

$$= \left[1 - \frac{e^{aT}}{z}\right]^{-1}$$

$$= \left[\frac{z-e^{aT}}{z}\right]^{-1}$$

$$\boxed{z[e^{at}] = \frac{z}{z-e^{aT}}}$$

6. Find the z- transform of e^{-at}

$$z[e^{-at}] = z[e^{-anT}]$$

$$= z[(e^{aT})^{-n}]$$

$$z\left[e^{-at}\right] = \frac{z}{z-e^{-aT}}$$

(or)

$$z\left[e^{-at}\right] = \sum_{n=0}^{\infty} e^{-anT} z^{-n} = \sum_{n=0}^{\infty} \frac{(e^{-aT})^n}{z^n}$$

$$= \sum_{n=0}^{\infty} \frac{z^{-n}}{e^{anT}} = \sum_{n=0}^{\infty} \left(\frac{e^{-aT}}{z}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{z}{e^{-aT}}\right)^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{e^{-aT}}{z}\right)^{n}$$

$$= 1 + \frac{e^{-aT}}{z} + \left(\frac{e^{-aT}}{z}\right)^2 + \cdots\cdots$$

$$= \left[1 - \frac{e^{-aT}}{z}\right]^{-1}$$

$$= \left[\frac{z-e^{-aT}}{z}\right]^{-1}$$

$$\boxed{z\left[e^{-at}\right] = \left[\frac{z}{z-e^{-aT}}\right]}$$

7. Find the z- transform of n.

$$z[n] = \sum_{n=0}^{\infty} n\, z^{-n}$$

$$= 1/z + \frac{2}{z^2} + 3/z^3 + \cdots\cdots$$

$$= 1/z \left[1 + \frac{2}{z} + 3/z^2 + \cdots\right]$$

$$= 1/z \left[1 - 1/z\right]^{-2}$$

$1 + 2x + 3x^2 + \cdots$
$= (1-x)^{-2}$

$$= \frac{1}{z}\left[\frac{z-1}{z}\right]^{-2}$$

$$= \frac{1}{\cancel{z}}\left[\frac{z^{\cancel{2}}}{(z-1)^2}\right]$$

$$\boxed{z[n] = \frac{z}{(z-1)^2}}$$

8. From the z - transform of a^{n-1}

$$z[a^{n-1}] = z^{-1} z[a^n]$$

$$= z^{-1}\frac{z}{z-a}$$

$$\boxed{z[a^{n-1}] = \frac{1}{z-a}.}$$

9. Find the z - transform of na^n

$$z[n\,f(n)] = -z\frac{d}{dz}\bar{f}(z)$$

$$z[n\,a^n] = -z\frac{d}{dz}\left[\frac{z}{z-a}\right]$$

$$= -z\left[\frac{(z-a)-z}{(z-a)^2}\right]$$

$$\boxed{z[na^n] = \frac{az}{(z-a)^2}}$$

10. Find the z - transform of n^2

$$z[n^2] = z[n \cdot \underline{n}]$$

$$= -z\frac{d}{dz}z[n]$$

$$= -z\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$= -z\left[\frac{(z-1)^2 - z \cdot 2(z-1)}{(z-1)^4}\right]$$

$$= -z(z-1)\left[\frac{(z-1)-2z}{(z-1)^4}\right]$$

$$= -z\left[\frac{-1-z}{(z-1)^3}\right]$$

$$\boxed{z[n^2] = \frac{z(z+1)}{(z-1)^3}}$$

11. Find the z- transform of $[n(n-1)]$

$$Z[n(n-1)] = z[n^2-n]$$

$$= z[n^2] - z[n]$$

$$= \frac{z(z+1)}{(z-1)^3} - \frac{z}{(z-1)^2}$$

$$= \frac{z(z+1) - z(z-1)}{(z-1)^3}$$

$$= \frac{z^2+z-(z^2-z)}{(z-1)^3}$$

$$\boxed{z[n(n-1)] = \frac{2z}{(z-1)^3}}$$

12. Find the z- transform of $[n(n+1)]$

$$z[n(n+1)] = z[n^2+n]$$

$$= z[n^2] + z[n]$$

$$= \frac{z(z+1)}{(z-1)^3} + \frac{z}{(z-1)^2}$$

$$= \frac{z(z+1) + z(z-1)}{(z-1)^3}$$

$$= \frac{z^2+z\ (z^2-z)}{(z-1)^3}$$

$$\boxed{z[n(n+1)] = \frac{2z^2}{(z-1)^3}}$$

Formula:

$$1/z + 1/2z^2 + 1/3z^3 + \dots = -\log(1-1/z)$$

(or)

$$x + x^2/2 + x^3/3 + \dots = -\log(1-x)$$

13. Find the z-transform of $1/n$

$$z[1/n] = \sum_{n=1}^{\infty} 1/n\ z^{-n}$$

$$= 1/z + 1/2z^2 + 1/3z^3 + \dots$$

$$= -\log(1-1/z)$$

$$= -\log\left(\frac{z-1}{z}\right)$$

$$= \log\left(\frac{z-1}{z}\right)^{-1}$$

$$\boxed{z(1/n) = \log\left(\frac{z}{z-1}\right)}$$

14. Find the z-transform of $1/n+1$

$$z\left[\frac{1}{n+1}\right] = \sum_{n=0}^{\infty} 1/n+1\ z^{-n}$$

$$= 1 + 1/2z + 1/3z^2 + \dots$$

$$= z[1/z + 1/2z^2 + 1/3z^3 + \dots]$$

$$= z[-\log(1-1/z)]$$

$$= z\left[-\log\left(\frac{z-1}{z}\right)\right]$$

$$= z\left[\log\left(\frac{z-1}{z}\right)^{-1}\right]$$

$$\boxed{z\left[\frac{1}{n+1}\right] = z\log\left(\frac{z}{z-1}\right)}$$

15. Find the z - transform of $1/n-1$

$$z\left[\frac{1}{n-1}\right] = \sum_{n=0}^{\infty} \frac{1}{n-1} z^{-n}$$

$$= -1 + 1/z^2 + \frac{1}{2z^3} + \frac{1}{3z^4} + \cdots$$

$$= -1 + 1/z\left[1/z + 1/2z^2 + 1/3z^3 + \cdots\right]$$

$$= -1 + 1/z\left[-\log(1 - 1/z)\right]$$

$$= -1 + 1/z \log\left(\frac{z}{z-1}\right)$$

$$\boxed{z\left[\frac{1}{n-1}\right] = 1/z \log\left(\frac{z}{z-1}\right) - 1}$$

16. Find the z - transform of $z\left[\frac{a^n}{n!}\right]$ and $\left[\frac{1}{n!}\right]$

$$z\left[\frac{a^n}{n!}\right] = \sum_{n=0}^{\infty} \frac{a^n}{n!} z^{-n}$$

$$= 1 + a/z \frac{1}{1!} + (a/z)^2 \, 1/2! + \cdots$$

$$= e^{a/z}$$

$$\boxed{z\left[\frac{a^n}{n!}\right] = e^{a/z}}$$

(ii) $Z\left[\frac{a^n}{n!}\right] = e^{a/z}$

put $a = 1$

$$\boxed{Z\left[\frac{1}{n!}\right] = e^{1/z}}$$

17. Find the z-transform of $\frac{1}{n+2}$

$$Z\left[\frac{1}{n+2}\right] = \sum_{n=0}^{\infty} \frac{1}{n+2} z^{-n}$$

$$= \frac{1}{2} + \frac{1}{3z} + \frac{1}{4z^2} + \frac{1}{5z^3} + \cdots$$

$$= z^2\left[\frac{1}{2z^2} + \frac{1}{3z^3} + \frac{1}{4z^4} + \cdots\right]$$

$$= z^2\left[\frac{1}{2z^2} + \frac{1}{3z^3} + \frac{1}{4z^4} + \cdots\right]$$

$$= z^2\left[\left(\frac{1}{z} + \frac{1}{2z^2} + \frac{1}{3z^3} + \frac{1}{4z^4} + \cdots\right) - \frac{1}{z}\right]$$

$$= z^2\left[-\log\left(1 - \frac{1}{z}\right) - \frac{1}{z}\right]$$

$$= z^2\left[\log\frac{z}{z-1} - \frac{1}{z}\right]$$

$$\boxed{Z\left[\frac{1}{n+2}\right] = z^2 \log\left(\frac{z}{z-1}\right) - z}$$

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1. $y_n = A2^n - B(-2)^n$ derive a difference eqn by Eliminating the constants.

$$y_n = a2^n - b(-2)^n \rightarrow (1)$$

$$y_{n+1} = a2^{n+1} - b(-2)^{n+1}$$

$$= 2a2^n + 2b(-2)^n \rightarrow (2)$$

$$y_{n+2} = a2^{n+2} - b(-2)^{n+2}$$

$$= 4a2^n + 4b(-2)^n \rightarrow (3)$$

To eliminate the A & B from (1), (2) & (3)

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 2 & -2 \\ y_{n+2} & 4 & 4 \end{vmatrix} = 0$$

$$y_n[8+8] - y_{n+1}(4-4) + y_{n+2}[-2-2] = 0.$$

$$16y_n - 4y_{n+2} = 0$$

$$4y_n - y_{n+2} = 0$$

2. From $y_n = a2^n + b(-2)^n$ derive a difference eqn by eliminating the constant.

$$y_n = a2^n + b(-2)^n \rightarrow (1)$$

$$y_{n+1} = a2^{n+1} + b(-2)^{n+1}$$

$$= 2a2^n - 2b(-2)^n \rightarrow (2)$$

$$y_{n+2} = a2^{n+2} + b(-2)^{n+2}$$

$$= 4a2^n + 4b(-2)^n \rightarrow (3)$$

To eliminate the A & B from (1),(2),(3)

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 2 & -2 \\ y_{n+2} & 4 & 4 \end{vmatrix} = 0$$

$y_n(8+8) - 1(4y_{n+1} + 2y_{n+2}) + 1(4y_{n+1} - 2y_{n+2}) = 0$

$16y_n - 4y_{n+1} - 2y_{n+2} + 4y_{n+1} - 2y_{n+2} = 0$

$16y_n - 4y_{n+2} = 0$

$4y_n - y_{n+2} = 0$ (Ans)

<u>Problems Based on Basic Transforms</u>

1. Find z transform of $(n+1)(n+2)$

$z[(n+1)(n+2)] = z[n^2 + 3n + 2]$

$= z[n^2] + 3z[n] + 2z[1]$

$= \frac{z(z+1)}{(z-1)^3} + 3\frac{z}{(z-1)^2} + 2\frac{z}{z-1}$

$= \frac{z^2+z}{(z-1)^3} + \frac{3z}{(z-1)^2} + \frac{2z}{z-1}$

$= \frac{z^2+z+3z(z-1)+2z(z-1)^2}{(z-1)^3}$

$= \frac{z^2+z+3z^2-3z+2z(z^2+1-2z)}{(z-1)^3}$

$$= \frac{\cancel{z^2} + \cancel{z} + 3\cancel{z^2} - 3\cancel{z} + 2z^3 - 4z^2 + 2z}{(z-1)^3}$$

$$= \frac{2z^3}{(z-1)^3} \text{ (Ans)}$$

2. find the z transform of $\frac{(n+1)(n+2)}{2}$

$$z\left[\frac{(n+1)(n+2)}{2}\right] = \frac{1}{2} z\left[n^2 + 3n + 2\right]$$

$$= \frac{1}{2}\left[\frac{2z^3}{(z-1)^3}\right]$$

$$= \frac{z^3}{(z-1)^3} \text{ (Ans)}$$

3. find the z transform of $z[n+1]$

$$z[n+1] = z[n] + z[1]$$

$$= \frac{z}{(z-1)^2} + \frac{z}{z-1}$$

$$= \frac{\cancel{z} + z^2 - z}{(z-1)^2}$$

$$= \frac{z^2}{(z-1)^2} \text{ (Ans)}$$

4. Find the z transform of $\frac{1}{n(n+1)}$

$$z\left[\frac{1}{n(n+1)}\right] = A/n + \frac{B}{n+1}$$

$$= A(n+1) + Bn$$

Put $n=-1$, $\boxed{1=-B}$

Put $n=0$, $\boxed{B=-1}$

$\boxed{A=1}$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$Z\left[\frac{1}{n(n+1)}\right] = Z[1/n] - Z[1/n+1]$$

$$= \log z/_{z-1} - z \log \frac{z}{z-1}$$

$$Z\left[\frac{1}{n(n+1)}\right] = (1-z) \log \frac{z}{z-1} \text{ (Ans)}$$

5 Find the z transform of $\frac{1}{n(n-1)}$

$$\frac{1}{n(n-1)} = \frac{A}{n} + \frac{B}{n-1}$$

$$= A(n-1) + Bn$$

Put $n=1$ Put $n=0$

$\boxed{B=1}$ $\boxed{A=-1}$

$$\frac{1}{n(n-1)} = \frac{-1}{n} + \frac{1}{n-1}$$

$$Z\left[\frac{1}{n(n-1)}\right] = Z[-1/n] + Z(1/n-1)$$

$$= -\log \frac{z}{z-1} + 1/z \log \frac{z}{z-1} - 1$$

$$= (1/z - 1) \log \frac{z}{z-1} - 1$$

$$Z\left[\frac{1}{n(n-1)}\right] = \left[\frac{1-z}{z}\right] \log \frac{z}{z-1} - 1 \quad \text{(Ans)}$$

6 find the Z transform of $\frac{2n+3}{(n+1)(n+2)}$

$$\frac{2n+3}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}$$

$$2n+3 = A(n+2) + B(n+1)$$

Put $n=-1$ put $n=-2$

$\boxed{1=A}$ $-B = -1$

$\boxed{B=1}$

$$Z\left[\frac{2n+3}{(n+1)(n+3)}\right] = Z\left[\frac{1}{n+1}\right] + Z\left[\frac{1}{n+2}\right]$$

$$Z\left[\frac{2n+3}{(n+1)(n+2)}\right] = z\log\left[\frac{z}{z-1}\right] + z^2\log\left[\frac{z}{z-1}\right] - z$$

$$= (z+z^2)\log\left[\frac{z}{z-1}\right] - z \quad \text{(Ans)}$$

7 find the Z transform of $u(n-1)$.

$$u(n-1) = \begin{cases} 1 & \text{if } n \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$Z[u(n-1)] = \sum_{n=0}^{\infty} u(n-1)\, z^{-n}$$

$$= \sum_{n=1}^{\infty} z^{-n}$$

$$= 1/z + 1/z^2 + \dots$$

$$= 1/z\left[1 + 1/z + 1/z^2 + \dots\right]$$

$$= 1/z\left[\frac{1}{1-1/z}\right]$$

$$= 1/z\left[\frac{z}{z-1}\right]$$

$$Z[u(n-1)] = \frac{1}{z-1} \quad \text{(Ans)}$$

8 Find the z transform of $\cos n\pi/2$.

$$f(n) = \cos n\pi/2$$

$$z[\cos n\pi/2] = \sum_{n=0}^{\infty} \cos n\pi/2 \, z^{-n}$$

$$= 1 - 1/z^2 + 1/z^4 - 1/z^6 + \ldots$$

$$= [1 + 1/z^2]^{-1}$$

$$= \left[\frac{z^2+1}{z^2}\right]^{-1}$$

$$z[\cos n\pi/2] = \frac{z^2}{z^2+1} \quad (\text{Ans})$$

9 Find the z transform of $\delta(n-k)$

$$\delta(n-k) = \begin{cases} 1 & \text{if } n=k \\ 0 & \text{otherwise} \end{cases}$$

$$z[\delta(n-k)] = \sum_{n=0}^{\infty} \delta(n-k) z^{-n}$$

$$= 1 \cdot z^{-k}$$

$$z[\delta(n-k)] = 1/z^k \quad (\text{Ans}).$$

10 Find the z transform of $3^n \delta(n-1)$

$$z[3^n \delta(n-1)] = z[\delta(n-1)]_{z \to z/3}$$

$$= [1/z]_{z \to z/3}$$

$$= 3/z \quad (\text{Ans})$$

11 find the z-transform of $2^n \sin n\pi/2$.

$$z\left[2^n \sin n\pi/2\right] = z\left[\sin n\pi/2\right]_{z \to z/2}$$

$$= \left[\frac{z}{z^2+1}\right]_{z \to z/2}$$

$$= \frac{z}{2}\left[\frac{1}{z^2/4+1}\right]$$

$$= \frac{z}{2}\left[\frac{4}{z^2+4}\right]$$

$$= \frac{2z}{z^2+4} \text{ (Ans)}$$

12 find the z-transform of $[2^n u(n-1)]$

$$z\left[2^n u(n-1)\right] = z\left[u(n-1)\right]_{z \to z/2}$$

$$= \left[\frac{1}{z-1}\right]_{z \to z/2}$$

$$= \frac{1}{z/2-1}$$

$$= \frac{2}{z-2} \text{ (Ans)}$$

Problem Based on Time Sequences

1. find the Z transform of t.

$$Z[t] = Z[nT]$$

$$= T[Z(n)]$$

$$= T\frac{z}{(z-1)^2}$$

$$Z[t] = \frac{Tz}{(z-1)^2} \quad \text{(Ans)}$$

[or]

$$Z[(t)] = \sum_{n=0}^{\infty} nT z^{-n}$$

$$= T/z + 2T/z^2 + 3T/z^3 + \dots$$

$$= T[1/z + 2/z^2 + 3/z^3 + \dots]$$

$$= T/z[1 + 1/z + 2/z^3 + \dots]$$

$$= T/z[1 - 1/z]^{-2}$$

$$= T/z\left[\frac{z-1}{z}\right]^{-2}$$

$$= T/z\left[\frac{z^2}{(z-1)^2}\right]$$

$$Z[t] = \frac{Tz}{(z-1)^2} \quad \text{(Ans)}$$

2. Find the z transform of t^2.

$$z[t^2] = z[n^2T^2]$$

$$= T^2 z[n^2]$$

$$= T^2 \frac{z(z+1)}{(z-1)^3} \quad (\text{Ans})$$

3. Find the z transform of t^3

$$z[t^3] = z[n^3T^3] \rightarrow (1)$$

$$z[n^3] = z[n\, n^2]$$

$$= -\frac{d}{dz} z[n^2]$$

$$= -\frac{d}{dz}\left[\frac{z(z+1)}{(z-1)^3}\right]$$

$$= -\left[\frac{(z-1)^3(2z+1) - (z^2+z)\,3(z-1)^2}{(z-1)^6}\right]$$

$$= -(z-1)^2\left[\frac{(z-1)(2z+1) - 3(z^2+z)}{(z-1)^6}\right]$$

$$= -\frac{1}{(z-1)^4}\left[2z^2 - 2z + z - 1 - 3z^2 - 3z\right]$$

$$= -\frac{1}{(z-1)^4}\left[-z^2 - 4z - 1\right]$$

$$z[n^3] = \frac{z^2+4z+1}{(z-1)^4} \rightarrow (2)$$

Sub (2) in (1)

$$z[t^3] = T^3 z[n^3]$$

$$z[n^3] = T^3\left[\frac{z^2+4z+1}{(z-1)^4}\right] \quad (\text{Ans})$$